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Counter-counter-intuitive quantum coherence effects

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The study of an ensemble of phase-coherent atoms has recently lead to interesting theoretical innovations and experimental demonstrations of counter-intuitive effects such as electromagnetically induced transparency (EIT), lasing without inversion (LWI), enhancement of index of refraction and ultra-large nonlinear susceptibility.

In the present notes, we report a couple of new effects along these lines. In fact, it would be fair to call them surprises even in the repertoire of 'counter-intuitive effects', i.e. 'counter-counter-intuitive effects'. Specifically, we will show that the LWI concept, which is based on *quantum* coherence, has an interesting counterpart in the *classical* physics of free electron laser operation.

In other current work, we find that it is possible to 'lock' atoms in an excited state via atomic coherence. It is, by now, not surprising that such a phase-coherent ensemble can show holes or dark lines in the emission spectrum. It *is* surprising that we can lock atoms in an excited (normally decaying) state via atomic coherence and interference. Such a phase-coherent collection of atoms, i.e. 'phaseonium', is indeed a novel new state of matter.

1. The free electron phaser (FEP): a phase-sensitive optical klystron

In a free-electron laser (FEL) (Madey 1971; Brau 1990; Dattoli *et al.* 1993), electrons are accelerated by a 'pondermotive potential' formed by the combined field of the wiggler and the laser, and this produces coherent stimulated radiation. Under the influence of the pondermotive potential, a grating in the spatial density of electrons ('bunching') on the scale of the laser wavelength is produced. As a result, net emission may be enhanced. If the current of electrons in the beam is high enough, or, equivalently, if the wiggler is long enough, bunching is significant and the gain is greatly increased; this leads to the 'large gain' regime (Bonifacio *et al.* 1984; Murphy *et al.* 1985).

A main limitation on gain is set by the spread in the longitudinal momentum of electrons in the beam. For this reason, much effort has been devoted to producing highly monoenergetic electron beams (Sessler *et al.* 1992; Yamazaki *et al.* 1993; Couprie *et al.* 1996).

Recently new approaches to the increase of gain in atomic lasers based on coherence and interference have lead to lasing without inversion (Kocharovskaya &

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Khanin 1988; Harris 1989; Scully *et al.* 1989; Boller *et al.* 1991; Hakuta *et al.* 1991; Kocharovskaya 1995; Zibrov *et al.* 1995; Padmabandu *et al.* 1996). This concept also has interesting implications for the FEL (a purely classical device!). It is conceptually implemented by an appropriate phasing of the electrons in the drift region between two wigglers via a static magnetic field.

Introduce the detuning Ω as the difference of electron momentum from resonant momentum

$$\Omega = \frac{k_{\mathrm{L}} + k_{\mathrm{W}}}{m\gamma^3}(p - p_{\mathrm{r}}),$$

where the resonant momentum is connected with phase velocity of the pondermotive potential

$$v_{\mathrm{r}} = \frac{\omega_{\mathrm{L}} - \omega_{\mathrm{W}}}{k_{\mathrm{L}} + k_{\mathrm{W}}},$$

as $p_{\mathrm{r}} = m\gamma_{\mathrm{r}} v_{\mathrm{r}}$ and the resonant Lorentz factor is

$$\gamma_{\mathrm{r}} = \frac{1}{\sqrt{1 - (v_{\mathrm{r}}^2/c^2)}},$$

where k_{W}, ω_{W} are wave vector and frequency of wiggler field, and k_{L}, ω_{L} are wave vector and effective frequency of laser field. The electron motion inside the wiggler depends on the phase of laser field at the moment the electron enters the ponderomotive potential. Due to motion in the drift region, the phase of the electron relative to the laser field changes: slow electrons (path 2) travel further due to the static B-field than do the fast ones (path 1). If Ω_0 is the detuning at the entrance of the first wiggler, we can express the phase-shift leading to cancellation of absorption for the electrons having $\Omega_0 < 0$ and to positive gain for $\Omega_0 > 0$ (Nikonov *et al.* 1996*a*,*b*, 1997) as

$$\Delta\phi = \begin{cases} \pi - \Omega L_{\mathrm{W}}, & \Omega_0 < 0, \\ -\Omega L_{\mathrm{W}}, & \Omega_0 > 0, \end{cases} \tag{1.1}$$

where L_{W} is the dimensionless wiggler length.

Phase-space motion (see figure 1) provides a way to achieve this cancellation. At the entrance of the first wiggler (figure 1*a*), we have electrons with the same detuning but different initial phases. The slow electrons acquire an extra π phase shift added by the drift region, which reverses the motion of the electron in the second wiggler. After the second wiggler (in figure 1*d*), there is no spread of electrons with negative detuning and consequently no absorption, but there is gain for electrons having positive detuning, i.e. we have LWI.

Let us remark that the realization of the phase-shift (1.1) is not trivial, because of its dependence on the entrance detuning, as it has been shown in Nikonov (1997). This can be achieved by oblique propagation of laser field to the wiggler axis (two-dimensional motion allows us to determine the initial detuning). This is referred to as a two-dimensional phased klystron, or free-electron phaser (FEP) (Harris 1996).

2. Quenching of spontaneous emission via atomic phase coherence

(*a*) *Driven four-level atom*

As a first example of a phase-coherent medium showing atoms 'locked' in an excited configuration, consider a four-level atom that consists of two upper levels $|a_1\rangle$ and

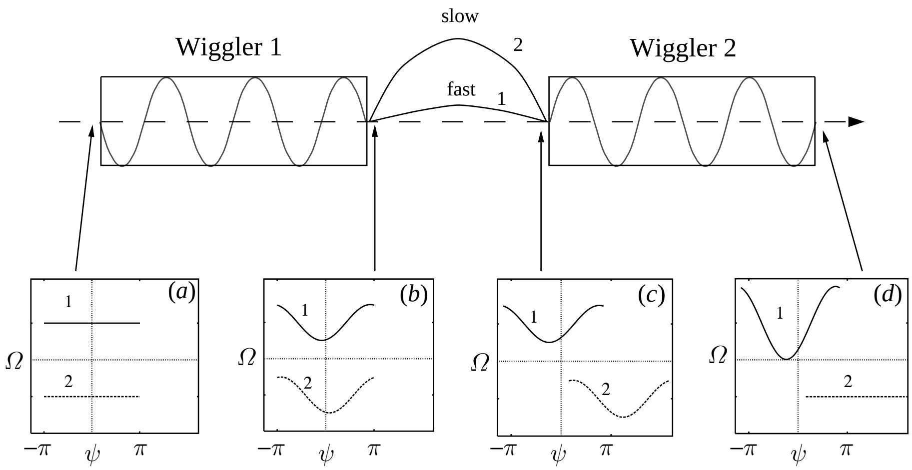


Figure 1. Snapshots of phase-space motion for fast (1) and slow (2) electrons. $\psi \equiv (k_{\mathrm{L}} + k_{\mathrm{W}})z - (\omega_{\mathrm{L}} - \omega_{\mathrm{W}})t + \phi$ is the total phase of an electron relative to the ponderomotive potential. (a) initial distribution of electrons before entering in wigglers; (b) after the first wiggler; (c) after the drift region (travelling in the static magnetic field the electrons pass along different paths depending on their detunings); (d) after the second wiggler, electrons with negative detuning return to their original energy states, and so do not absorb energy from the field. Fast electrons, however, do lose energy to the field; therefore, there is only gain.

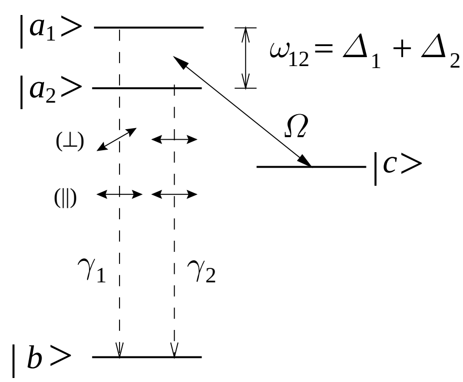


Figure 2. Two upper levels $|a_1\rangle$ and $|a_2\rangle$ are coupled to the ground state $|b\rangle$ by vacuum modes and to an auxilliary level $|c\rangle$ by a coherent drive. The dipole elements for the $a \rightarrow c$ transitions may be either perpendicular ($\perp$) or parallel ($\|$). Ω is the drive field, ω_{12} is the frequency difference between $|a_1\rangle$ and $|a_2\rangle$, and Δ_1 and Δ_2 are the detunings of the drive from $|a_1\rangle$ and $|a_2\rangle$, respectively.

$|a_2\rangle$, and an intermediate level $|c\rangle$ (Zhu & Scully 1996; Xia *et al.* 1996). The two upper levels are coupled by the vacuum to the lower level $|b\rangle$. If the dipole elements for the two transitions are parallel, they are coupled by the same vacuum modes (see figure 2).

The spontaneous emission spectra for orthogonal ($\perp$) and parallel ($\|$) dipole elements (figure 2) are quite different due to quantum interference. It is well known that the spontaneous emission spectrum for the orthogonal ($\perp$) case is a three-peak distribution. For parallel ($\|$), however, we have interference, which can lead to the elimination of one of the three peaks. In figure 3, we plot spectra for both cases with the atom initially in $|a_1\rangle$. We see the disappearance of the central peak as a result

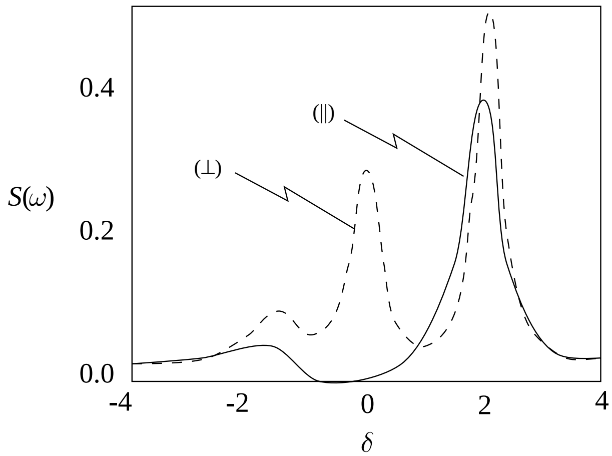
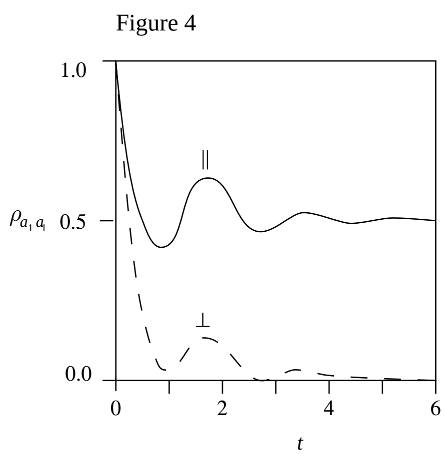


Figure 3. Spontaneous emission spectra for orthogonal ($\perp$) and parallel ($\|$) transition dipole matrix elements. The atom is initially in state $|a_1\rangle$ (figure 2).



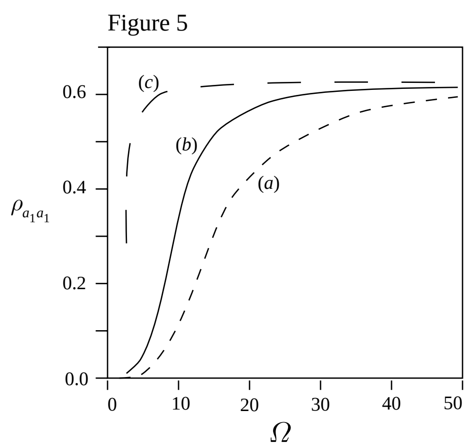


Figure 4. Time evolution of population in $|a_1\rangle$ (see figure 3) for orthogonal ($\perp$) and parallel ($\|$) dipole moments.
Figure 5. Trapped population in level $|a_1\rangle$ (see figure 3) versus drive Rabi frequency Ω for various values of ω_{12}: (a) $40\gamma_1$, (b) $20\gamma_1$ and (c) $4\gamma_1$.

of interference†. The elimination of the central peak indicates the cancellation of the spontaneous emission into those modes with frequencies near the central peak (in the neighbourhood of the driving field frequency) and is the result of destructive interference.

The area under the spectral curve is proportional to the energy emitted by the atom into the vacuum modes. For orthogonal polarizations, the area is always equal to unity (energy conservation); that is, the atom will finish in the lower level $|b\rangle$. For parallel polarization, we find that the area may be less that unity, and we may

† It can be proven analytically that the central peak is eliminated if $\Delta_2 = -\chi^2\Delta_1$, where $\chi = |\Omega_2/\Omega_1| = g^{(2)}/g^{(1)}$ is the ratio of dipole moments between the upper two levels and level $|c\rangle$, which is also equal to the ratio of the dipole moments between the two upper levels and level $|b\rangle$ (see Zhu & Scully 1996).

conclude on this basis that spontaneous emission in this case is cancelled by quantum interference, i.e. the atom can live forever in the excited state.

In figure 4, we plot the evolution of the population in level $|a_1\rangle$ for both polarizations. It is clear that this population goes to zero for orthogonal, and *ca.* 50% for parallel polarizations. Cancellation of spontaneous emission in the steady state is another example of quantum interference.

In figure 5, we plot the population in level $|a_1\rangle$ versus the Rabi frequency for three cases with $\chi = 2$ (see footnote on previous page). It can be seen that 30% of the population will be trapped in level $|a_1\rangle$ if the Rabi frequency is $10\gamma_1$ for a large separation ($\omega_{12} = 40\gamma_1$). In order to trap more population, we need a higher Rabi frequency.

Spectral truncation and cancellation of spontaneous emission can be understood in a dressed-state picture. On diagonalizing the Hamiltonian for $|a_1\rangle$ and $|a_2\rangle$, $|b\rangle$ and the driving field, we get three dressed states. The decay from $|a_1\rangle$ and $|a_2\rangle$ to $|b\rangle$ becomes the decay from the dressed states to $|b\rangle$. The interference in the parallel ($\|$) case results in a zero decay rate. The population in one of the dressed states will not decay to lower level $|b\rangle$, and consequently we have central peak elimination and spontaneous emission cancellation in steady state.

(*b*) *Three-level atom (no drive) in photonic band-gap material*

Now let us consider the case of a three-level atom (figure 6) embedded in a 'photonic crystal' (John & Wang 1990, 1991; John & Quang 1994). Assume the edge of the photonic band gap is located between levels $|a_1\rangle$ and $|a_2\rangle$. The dispersion relation near the band edge can be approximately written as $\omega = \omega_c + A(k - k_0)^2$. Therefore, level $|a_2\rangle$ is within the band gap, while level $|a_1\rangle$ is in the band. In the band gap, the density of states is zero. There is a singularity in the density of states at $\omega = 0$. For $\omega > \omega_c$, the density of states decreases. The spontaneous emission of an atom embedded in a photonic band gap crystal is quite different from the spontaneous emission in a vacuum. Population can be trapped in the two upper levels due to (a) the photonic band gap, and (b) the interference between the two transitions. With interference, we can trap more population in the two upper levels if we prepare the atom in a special initial state. One might think that the largest amount of population would be trapped in the upper levels if the atom were initially prepared in the level within the gap (level $|a_2\rangle$) because of the low density of states. In fact, this is not the case.

The amount of population trapped in the upper levels depends on the initial condition. The population in the upper level within the band gap ($|a_2\rangle$) can be transferred to the upper level in the transmitting band ($|a_1\rangle$) via emission and reabsorption of a photon. The final state contains an upper level part (trapped excited state population) and a lower level part with one photon in the localized mode. The phase difference between the two upper levels in the final state turns out to be zero. The dependence of the ratio of the populations in the upper two levels, $A^{(1)}(\infty)/A^{(2)}(\infty)$ on the initial state is weak. This ratio, as well as the portion in the lower level, depends on Δ/γ.

Since the true trapped final state is a dressed state with a lower-level component, no superposition of the upper levels can evade decay completely. This is in contrast with the dark state of a driven four-level system. However, if the atom is prepared in a state which is a normalized version of this final state projected onto the manifold

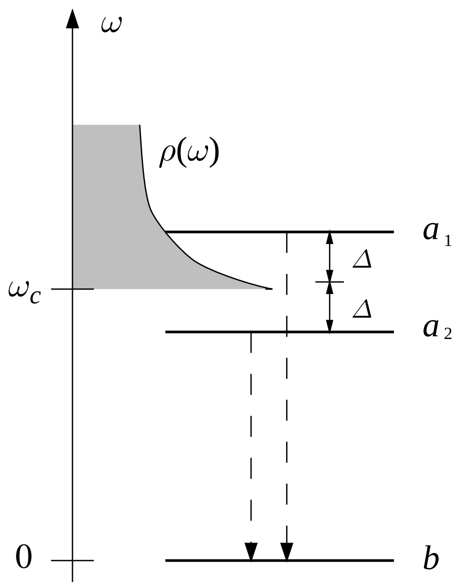


Figure 6. The scheme: a three-level atom in a photonic band-gap structure (represented by the grey-shaded curve). The upper levels $|a_1\rangle$ and $|a_2\rangle$ are placed symmetrically from the band gap edge.

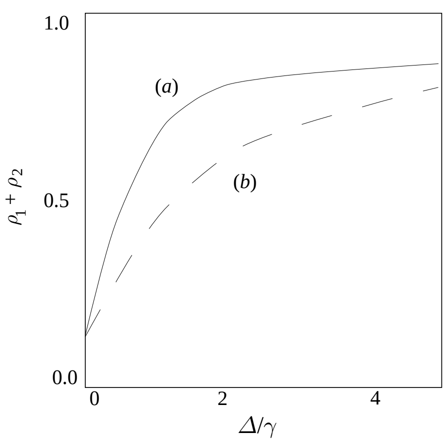


Figure 7. Sum of populations in states $|a_1\rangle$ and $|a_2\rangle$ as a function of Δ. (a) initial condition given by equation (2.1), (b) $|\psi(0)\rangle = |a_2\rangle$.

of the two upper levels

$$|\psi(0)\rangle = \frac{A^{(1)}(\infty)|a_1\rangle + A^{(2)}(\infty)|a_2\rangle}{\sqrt{|A^{(1)}|^2 + |A^{(2)}|^2}}, \tag{2.1}$$

we can minimize the energy emitted and have more population trapped in the upper levels, even more than the case with the atom in the level in the gap ($|a_2\rangle$), due to quantum interference.

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